THE BATES AND SCOTT MODELS

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Abstract

This article reviews the Bates and Scott option pricing models, which capture both stochastic volatility and jump risk within a tractable affine specification.

Key words

options, affine models, Fourier inversion, jumps, stochastic volatility, Bates model, Scott model

The Bates [3] and Scott [13] option pricing models were designed to capture two features of asset returns: the fact that conditional volatility evolves over time in a stochastic but mean-reverting fashion, and the presence of occasional substantial outliers in asset returns. The two models combined the Heston [9] model of stochastic volatility with the Merton [11] model of independent normally distributed jumps in the log asset price. The Bates model ignores interest rate risk, while the Scott model allows interest rates to be stochastic. Both models evaluate European option prices numerically, using the Fourier inversion approach of Heston. The Bates model also includes an approximation for pricing American options. The two models were historically important in showing that the tractable class of affine option pricing models includes jump processes as well as diffusion processes.

All option pricing models rely upon a risk-neutral representation of the data generating process that includes appropriate compensation for the various risks. In the Bates and Scott models, the risk-neutral processes for the underlying asset price S_t and instantaneous variance V_t are assumed to be of the form

$$\begin{aligned} dS_t/S_t &= (b - \lambda^* \bar{k}^*)\,dt + \sqrt{V_t}\,dZ_t + k^*\,dq_t \\ dV_t &= (\alpha - \beta^* V_t)\,dt + \sigma_v \sqrt{V_t}\,dZ_{vt} \end{aligned} \qquad \textbf{(1)}$$

where b is the cost of carry; Z_t and Z_{vt} are Wiener processes with correlation ρ; q_t is an integer-valued Poisson counter with risk-neutral intensity λ^* that counts the occurrence of jumps; and k^* is the random percentage jump size, with a Gaussian distribution $\ln(1+k^*) \sim N[\ln(1+\bar{k}^*) - \tfrac{1}{2}\delta^2,\ \delta^2]$ conditional upon a jump occurring. The Bates model assumes b is constant, while the Scott model assumes it is a linear combination of V_t and an additional state variable that follows an independent square-root process. Bates [3] examines foreign currency options, for which b is the domestic/foreign interest differential, while Scott's application [13] to non-dividend paying stock options implies the cost of carry is equal to the risk-free interest rate.

The postulated process has an associated conditional characteristic function that is exponentially affine in the state variables. For the Bates model, the characteristic function is

$$\begin{aligned} \varphi(i\Phi) &\equiv E_0^*\left[e^{i\Phi \ln S_T} \mid S_0, V_0, T\right] \\ &= \exp\left[i\Phi S_0 + C(T; i\Phi) + D(T; i\Phi)\,V_0 + \lambda^* T E(i\Phi)\right] \end{aligned} \qquad \textbf{(2)}$$

where $E_0^*[\cdot]$ is the risk-neutral expectational operator associated with equation (1), and

$$\gamma(z) \equiv \sqrt{(\rho\sigma_v z - \beta)^2 - \sigma_v^2(z^2 - z)}$$

$$C(T;z) = bTz - \frac{\alpha T}{\sigma_v^2}\left[\rho\sigma_v z - \beta^* - \gamma(z)\right] - \frac{2\alpha}{\sigma_v^2}\ln\left[1 + \left[\rho\sigma_v z - \beta^* - \gamma(z)\right]\frac{1 - e^{\gamma(z)T}}{2\gamma(z)}\right]$$

$$D(T;z) = \frac{z^2 - z}{\gamma(z)\dfrac{e^{\gamma(z)T} + 1}{e^{\gamma(z)T} - 1} + \beta^* - \rho\sigma_v z}$$

$$E(z) = (1 + \bar{k}^*)^z e^{½\delta^2(z^2 - z)} - 1 - \bar{k}^* z.$$

The terms $C(\cdot)$ and $D(\cdot)$ are identical to those in the Heston [9] stochastic volatility model, while $E(\cdot)$ captures the additional distributional impact of jumps. Scott's generalization to stochastic interest rates uses an extended Fourier transform of the form

$$\varphi^*(z) \equiv E_0^*\left[\exp\left(-\int_0^T r_t\,dt + z\ln S_T\right) \mid S_0, r_0, V_0, T\right] \tag{3}$$

which has an analytical solution for complex-valued z that is also exponentially affine in the state variables S_0, r_0 and V_0.

European call option prices take the form $c = B(FP_1 - XP_2)$, where B is the price of a discount bond of maturity T, F is the forward price on the underlying asset, X is the option's exercise price, and P_1 and P_2 are upper tail probability measures derivable from the characteristic function. The Bates (1996) and Scott (1997) papers present Fourier inversion methods for evaluating P_1 and P_2 numerically. However, faster methods were subsequently developed for directly evaluating European call option prices, using a single numerical integration of the form

$$c = BF - BX\left\{\frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{f(i\Phi)e^{-i\Phi\ln X}}{i\Phi(1 - i\Phi)}\right]d\Phi\right\} \tag{4}$$

where $Re[z]$ is the real component of a complex variable z. For the Bates model, $f(i\Phi) = \varphi(i\Phi)$; for the Scott model, $f(i\Phi) = \varphi^*(i\Phi)/B$. European put options can be evaluated from European call option prices using the put-call parity relationship $p = c + B(X - F)$.

Evaluating equation (4) typically involves integration of an dampened oscillatory function. While there exist canned programs for integration over a semi-infinite domain, most papers use various forms of integration over a truncated domain. Bates [3] uses Gauss-Kronrod quadrature. Fast Fourier

Transform approaches have also been proposed, but involve substantially more functional evaluations. The integration is typically well-behaved, but there do exist extreme parameter values (e.g., $|\rho|$ near 1) for which the path of integration crosses the branch cut of the log function. As all contemporaneous option prices of a given maturity use the same values of $f(i\Phi)$ regardless of the strike price X, evaluating options jointly greatly increases numerical efficiency

Related models

Related affine models can be categorized along four lines:

1) alternate specifications of jump processes;

2) the Bates [5] extension to stochastic-intensity jump processes;

3) models in which the underlying volatility can also jump; and

4) multifactor specifications.

Alternate jump specifications (including Lévy processes) with independent and identically distributed jumps involve modification of the functional form of $E(\cdot)$, and are discussed in other articles in this *Encyclopedia*. The Bates [5] model with (risk-neutral) stochastic jump intensities of the form $\lambda^* + \lambda_1^* V_t$ involves modifying $\gamma(\cdot)$ and $D(\cdot)$:

$$\gamma(z) \equiv \sqrt{(\rho\sigma_v z - \beta)^2 - \sigma_v^2[z^2 - z + 2\lambda_1^* E(z)]}$$

$$D(T;z) = \frac{z^2 - z + 2\lambda_1^* E(z)}{\gamma(z)\dfrac{e^{\gamma(z)T}+1}{e^{\gamma(z)T}-1} + \beta^* - \rho\sigma_v z}.$$

Bates [5] also contains multifactor specifications for the instantaneous variance and jump intensity. The general class of affine jump-diffusion models is presented in Duffie et al [8], including the volatility-jump option pricing model. Scott's extended Fourier transform approach for stochastic interest rates was subsequently also used by Bakshi and Madan [2] and Duffie et al.

Further reading

Bates [7, pp. 943-4] presents a simple derivation of equation (4), and cites earlier papers that develop the single-integration approach. Numerical integration issues are discussed in Lee [10]. Bates [3] and Bakshi et al [1] estimate and test the Bates and Scott models, respectively, while Pan [12] provides additional estimates and tests of the Bates [5] stochastic-intensity model. Bates [4, 6] surveys empirical option pricing research.

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