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The authors derive a theoretical relationship between the aggregate market share elasticity matrix and the aggregate brand switching matrix on the basis of a logit model of heterogeneous consumers choosing among competing brands in a product class. Aggregate cross-elasticities are shown to be proportional (through a single scaling constant) to their corresponding aggregate row-conditional brand switching probabilities. Aggregate own-elasticities are shown to be proportional (through the negative of the same scaling constant) to one minus their corresponding aggregate row-conditional repeat purchase probabilities. An empirical analysis conducted on household scanner panel data in the liquid laundry detergent category shows that the theoretical correspondence holds as a very good approximation. An illustrative use of the relationship in estimating aggregate (store-level) models of market share indicates that the relationship helps improve predictive validity in a holdout period.

A Relationship Between Market Share Elasticities and Brand Switching Probabilities

Measures of substitutability among competing brands are indispensable to marketing researchers and managers seeking to understand better the differential levels of competition among multiple brands in a product category. For example, in tactical competitive analysis, managers are interested in quantifying the extent to which price changes by one brand affect another brand.

Two of the most commonly used measures of interbrand substitutability are cross-elasticities and brand switching probabilities. Cross-elasticities can be estimated by logit models calibrated on household-level panel data (e.g., Guadagni and Little 1983; Kamakura and Russell 1989) or by econometric methods on aggregate, store-level data (e.g., Allenby 1989; Cooper 1988); survey-based approaches also have been developed (Bordley 1993; Bucklin and Srinivasan 1991). Brand switching probabilities typically are estimated from panel or survey data either as cross-classification probabilities (proportion of times brands i and j are purchased on two adjacent occasions) or as row-conditional switching probabilities (of those who purchased brand i last time, the proportion purchasing brand j during the next purchase occasion).

The two measures of substitutability lead to the intuitive notion that brand switching and cross-elasticity matrices contain the same fundamental information and therefore may correspond in a theoretical fashion. To the best of our knowledge, however, no formal link has been made yet between the measure of substitutability traditionally used by economists (elasticity) and the measure of substitutability traditionally used by marketers (switching probability). Ambiguity also exists about the more appropriate measure of brand switching because some investigators have worked with cross-classification probabilities, whereas others have used row-conditional switching probabilities. A direct mathematical correspondence between elasticities and one of these measures of brand switching therefore would be of theoretical interest.

Elasticities have direct managerial value in terms of assessing the impact of the firm's and competitors' marketing actions on market shares. Often elasticity estimates are difficult to obtain because of either insufficient data or problems with estimation. For example, if household-level scanner panel data are available for the product category and the geographic market of interest, logit models can be used

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to estimate elasticities (as is done in the empirical section of this article). If the household-level scanner panel data are collected by consumers' use of "wands," the information on the store environment (e.g., shelf prices and promotional activity) is often incomplete, thereby introducing substantial potential error into the logit analysis. Furthermore, household-level scanner panel data are unavailable or only partially available in many product categories (e.g., fast food, camera film, motor oil). Even when such data are available for some geographic areas, they may be unavailable for others. For these reasons, researchers frequently must estimate elasticities on the basis of aggregate store-level data using econometric methods. A major difficulty with such aggregate-level analyses is that econometric issues, such as multicollinearity, make the elasticity estimates considerably error-prone (e.g., the aggregate market share model in the empirical section leads to 38% of the cross-elasticities having the incorrect [negative] sign).

Although brand switching probabilities do not have the direct managerial value that elasticities have, they can be assessed more accurately. Switching probabilities are always non-negative and therefore have the correct sign. Moreover, brand switching probabilities always can be obtained from survey data. Thus, in addition to theoretical interest in a mathematical relationship between elasticities and brand switching probabilities, an important practical benefit is that researchers and managers can obtain the more useful elasticities from the more accurately measurable brand switching probabilities.

Our objective is to formalize a theoretical relationship between elasticities and brand switching probabilities, provide empirical evidence supporting this relationship, and illustrate its use in estimating aggregate market share models. Given some plausible assumptions about consumer choice behavior (detailed subsequently), we derive the following relationships:

- Aggregate market share cross-elasticities are proportional (through a single scaling constant) to their corresponding aggregate row-conditional switching probabilities.
- Aggregate market share own-elasticities are proportional (through the negative of the same scaling constant) to one minus their corresponding aggregate row-conditional repeat purchase probabilities.

The scaling constant would, in general, be different for different marketing mix elements (e.g., price, feature, display).

Research Approach

Our derivation of the aggregate switching–elasticity relationship for a population of heterogeneous consumers proceeds in two phases. The first phase derives the relationship for a hypothetical short time period during which the marketing environment, that is, brand prices and other marketing activity, is assumed to remain essentially fixed. The second phase generalizes the relationship for a longer time horizon during which the marketing environment fluctuates from period to period. An expositional advantage of the two-phase derivation is that the switching–elasticity relationship is derived easily with merely the first phase. The second phase derives the relationship under more realistic conditions. It does not change the nature of the relationship but changes primarily the scaling constant.

We emphasize that the theoretical relationships derived in this article are conditional on the logit model (i.e., the exact correspondence does not hold, in general, for other choice models). Nevertheless, the logit model has proven to be empirically quite robust. Consequently, we believe that the empirical correspondence implied by the theory will hold as a good approximation in many markets. In particular, we show that the empirical correspondence holds up under the full covariance probit model, which does not suffer from the independence from irrelevant alternatives (IIA) assumption of the logit model.

Intuitively, we are able to derive a relationship between elasticities and brand switching probabilities for the following reasons:

- For a consumer whose choices are consistent with the multinomial logit model, elasticities are merely functions of brand choice probabilities. This is true even though the probabilities, in turn, depend on intrinsic brand preferences, prices, and other marketing variables. In other words, when we know the brand choice probabilities, we can determine elasticities (up to a scaling coefficient) without any knowledge of the values of the underlying causal variables that determine the choice probabilities.
- A consumer's brand switching is merely a function of choice probabilities.
- The relationship between elasticities and brand switching probabilities is obtained because both measures depend in a similar manner on brand choice probabilities.

In the empirical section, we evaluate the theoretical relationship on scanner data for the liquid laundry detergent category. We assess the extent to which aggregated row-conditional brand switching probabilities relate to market share own- and cross-elasticities as predicted by the theory. Using results from a logit model, we assess the relationship for three marketing variables: price, feature, and display. We also assess the empirical correspondence with price elasticities computed from (1) a full covariance probit model and (2) a multisegment logit model. In all cases, we find the empirical correspondence to hold as a good approximation. Finally, we demonstrate the potential use of the relationship in enhancing the predictive validity of an aggregate (store-level) market share model.

CONSUMER PREFERENCE AND CHOICE

We begin our characterization of consumer purchase behavior with a random utility model to capture the utility U that consumer k assigns to brand i at time t:

$$U_{ikt} = u_{ik} + \sum_{l} v_l \ln(x_{ilt}) - v_p \ln(p_{it}) + z_{ikt}. \tag{1}$$

Here, u_{ik} is the intrinsic preference for brand i for consumer k. Note that we allow for heterogeneity in intrinsic preference across brands and consumers. The term $\sum_l v_l \ln(x_{ilt})$ captures the net effect of the nonprice marketing variables (indexed by l) $\{x_{ilt}\}$ for brand i at time t, with the effects determined by the coefficients $\{v_l\}$. The term $v_p \ln(p_{it})$ captures the effects of price p_{it} for brand i. Our specification of utility in terms of the log of marketing variables implies that consumer preference varies with percentage changes in marketing activity rather than absolute changes (Allenby 1989). Such a specification is consistent with the literature on just-noticeable differences and the Weber-Fechner law of psychophysics (e.g., Monroe 1979, p. 42; Torgerson 1958, p. 149).[1,2] The term z_{ikt} denotes random error.

Equation 1 permits an unrestricted amount of heterogeneity across consumers in terms of intrinsic brand preferences. However, the coefficients for the (log of) marketing variables are assumed to be common across consumers. At first, it might appear that we have made the unrealistic assumption that sensitivity of utility to marketing activity is identical across consumers. Such is not the case. For expositional ease, we ignore the nonprice marketing variables, the random error term, and the time subscript so that total utility is given by $u_{ik} - v_p \ln(p_i)$. For example, consumers A and B might both prefer brand 1 over brand 2, but consumer A might have a much smaller difference (spread) in preference u across the two brands compared with consumer B. Suppose initially that the two brands are priced equally so that both consumers prefer brand 1 to brand 2 on an overall basis. Then, a reduction in the price of brand 2 might make consumer A but not consumer B switch his or her overall preference from brand 1 to brand 2. Thus, in this model, price-sensitive consumers will have smaller absolute differences in the u_{ik} term across brands than price-insensitive consumers. A similar line of analysis applied to the nonprice marketing variables indicates that customers with small absolute differences in the u_{ik} term across brands will be more sensitive to nonprice marketing variables. Thus, in this model, a customer who is more responsive to one marketing variable is also more responsive to other marketing variables.[3]

We recognize that Equation 1 does not permit the form of heterogeneity in which consumer A is more responsive to one marketing variable but consumer B is more responsive to some other marketing variable. We hope that further research will generalize our results by including a greater amount of heterogeneity in the response coefficients as in Gonul and Srinivasan's (1993), Jain, Vilcassim, and Chintagunta's (1994), and Kamakura and Russell's (1989) studies. To summarize, Equation 1 provides a parsimonious utility model that permits an unrestricted amount of heterogeneity in intrinsic brand preferences but only a limited amount of heterogeneity in sensitivity to marketing variables.

[1]If Equation 1 were modified to be linear in marketing variables (rather than log-linear), the right-hand side of the theoretical relationships in Equations 20 and 21 for price elasticities would be augmented by multiplicative terms p_i and p_j, respectively, denoting the prices prevailing during the short time period under consideration. The same modifications would apply to Equations 32 and 33, in which p_i and p_j refer to average prices over the time horizon considered. In the empirical application reported subsequently, the fit (log-likelihood) was nearly identical for the logarithmic price term and the linear price term.

[2]A potential problem with a derivation based on the logarithm of marketing variables is that some marketing variables may be dichotomous, that is, equal to zero or one. This can be handled by resetting zeroes to one and ones to e.

[3]Mathematically, Equation 1 for consumer k can be modified to

$$\tilde{U}_{ikt} = \tilde{u}_{ik} + \sum_l (a_k v_l)\ln(x_{ilt}) - (a_k v_p)\ln(p_{it}) + \tilde{z}_{ikt},$$

where a_k is a consumer-specific parameter. Dividing the equation throughout by a_k, we obtain the utility function in the form of Equation 1:

$$U_{ikt} = u_{ik} + \sum_l v_l \ln(x_{ilt}) - v_p \ln(p_{it}) + z_{ikt},$$

where $U_{ikt} = \tilde{U}_{ikt}/a_k$, $u_{ik} = \tilde{u}_{ik}/a_k$, and $z_{ikt} = \tilde{z}_{ikt}/a_k$.

Following the assumptions of the logit brand choice modeling literature (e.g., Chintagunta, Jain, and Vilcassim 1991; Guadagni and Little 1983), we assume that the random error z_{ikt} is distributed independently and identically across brands, consumers,[4] and purchase occasions according to the type-I extreme value distribution:

$$\text{(2)} \qquad \text{Prob}\{z_{ikt} \le z_0\} = \exp[-\exp(-\mu z_0)],$$

with variance

$$\text{(3)} \qquad \text{Var}(z_{ikt}) = \pi^2/6\mu^2,$$

so that $\mu > 0$ is inversely related to the variance of the error term z_{ikt}. Thus, as $\mu \to 0$, the error variance becomes extremely large so that brand choices are mostly random and the choice probabilities become nearly equal across brands.

The probability θ_{ikt} that consumer k chooses brand i at time t is given by the probability that $U_{ikt} > U_{skt}$ for all s, where s denotes any brand other than i. Thus, the probability θ_{ikt} is given by the well-known logit model

$$\text{(4)} \qquad \theta_{ikt} = \frac{\exp\left[u'_{ik} + \sum_l \beta_l \ln(x_{ilt}) - \beta_p \ln p_{it}\right]}{\sum_s \exp\left[u'_{sk} + \sum_l \beta_l \ln(x_{slt}) - \beta_p \ln p_{st}\right]},$$

where $u'_{ik} = \mu u_{ik}$, $\beta_l = \mu v_l$, and $\beta_p = \mu v_p$. Thus, β_p and $\{\beta_l\}$ reflect the impact on choice probabilities of price and nonprice marketing variables, respectively.

THE RELATIONSHIP UNDER FIXED MARKETING CONDITIONS

This section develops the relationship between aggregate elasticities and aggregate brand switching over a hypothetical short time period during which all marketing activity remains essentially fixed. Consequently, our development in this section suppresses the time subscript t. (The intertemporal generalization in which prices and marketing activity fluctuate from period to period is considered in the next section.) Our derivations are based on point elasticities, in which changes in marketing variables are infinitesimal. (Arc elasticities, in contrast, are computed over a discrete change in the marketing variable of interest.) For expositional ease, we base the development that follows on price and price elasticities, noting that parallel derivations also apply to nonprice marketing variables. We begin by considering choice elasticities at the consumer level and then aggregate across consumers to generate aggregate market share elasticities.

Individual and Aggregate Elasticities

The own-price elasticity e_{iik} for brand i and the cross-price choice elasticity e_{ijk} of a change in price by brand j on brand i's choice probability for a specific consumer k for the logit model are the well-known expressions (e.g., Ben-Akiva and Lerman 1985)[5]

$$\text{(5)} \qquad e_{iik} = -\beta_p(1 - \theta_{ik}) \text{ and}$$

[4]The results in this article are unaffected if the error term is correlated across consumers.

[5]The elasticities are defined as $e_{iik} = (\partial\theta_{ik}/\partial p_i)(p_i/\theta_{ik})$ and $e_{ijk} = (\partial\theta_{ik}/\partial p_j)(p_j/\theta_{ik})$. Thus, in the price elasticity matrix, e_{ij} denotes the effect on row brand i of column brand j's price change.

$$e_{ijk} = \beta_p \theta_{jk} \text{ for } i \neq j. \tag{6}$$

We note that intrinsic brand preferences u, nonprice marketing activity $\{x_l\}$, and prices p do not appear explicitly in the elasticity expressions; their effects, however, are captured indirectly through the choice probabilities θ. This key feature of the logit model—that the elasticities depend only on the summary measure of choice probability and not directly on the underlying causative components—enables us to relate elasticity to brand switching. As is shown subsequently, brand switching also is related to choice probabilities.

Equations 5 and 6 show that choice elasticities can be quite different across consumers and brands. Consumers are relatively insensitive to price changes on brands with high purchase probabilities (Equation 5). Moreover, brands with low purchase probabilities would have little success in inducing switches using a price change (Equation 6). Thus, differences in a consumer's choice elasticities reflect differences in the choice probabilities that in turn are based on intrinsic brand preferences, nonprice marketing activities, and the prevailing set of brand prices. The proportionality constant β_p reflects the notion that if the effect of a price change on choice probability is large (Equation 4), then price elasticities will be large.

To aggregate the individual-level choice elasticities to market share elasticities, we define the market share of brand i as

$$MS_i = \frac{\left(\sum_k \theta_{ik}\right)}{N}, \tag{7}$$

where N is the total number of consumers in the market. For expositional simplicity, we assume that product category purchase quantities (assumed to be exogenous) are the same across consumers, so that there is no need to weight consumers differentially. Nevertheless, it easily is shown that the obtained theoretical relationships between elasticities and brand switching probabilities are unaffected if we define market shares taking into account the differential purchase quantities across consumers.[6] We define the market share elasticity as

$$e_{ij} = \frac{\partial MS_i}{\partial p_j} \times \frac{p_j}{MS_i}. \tag{8}$$

Using the previous definition of market share MS_i, it follows that

$$e_{ii} = \frac{\sum_k \theta_{ik} e_{iik}}{\sum_k \theta_{ik}} \tag{9}$$

and

$$e_{ij} = \frac{\sum_k \theta_{ik} e_{ijk}}{\sum_k \theta_{ik}} \text{ for } i \neq j. \tag{10}$$

[6]Let q_k denote consumer k's purchase quantity. Then, Equations 12 and 13 and Equations 18 and 19 are modified in both the numerator and denominator by the inclusion of a multiplicative term q_k inside the summations. Thus, Equations 20 and 21 and Equations 32 and 33 are unaffected.

Thus, the market-level elasticities are simply weighted averages of the consumer choice elasticities (Ben-Akiva and Lerman 1985) with weights

$$w_{ik} = \frac{\theta_{ik}}{\sum_k \theta_{ik}}. \tag{11}$$

The weight w_{ik} reflects consumer k's importance in determining brand i's share. *Thus, the consumer weights for brand i are, in general, different from those for brand j.*

Substituting from Equations 5 and 6, the market-level price elasticities are given by

$$e_{ii} = \frac{-\beta_p \sum_k \theta_{ik}(1-\theta_{ik})}{\sum_k \theta_{ik}} = -\beta_p \left(1 - \frac{\sum_k \theta_{ik}\theta_{ik}}{\sum_k \theta_{ik}}\right) \tag{12}$$

and

$$e_{ij} = \frac{\beta_p \sum_k \theta_{ik}\theta_{jk}}{\sum_k \theta_{ik}} \text{ for } i \neq j. \tag{13}$$

The elasticities satisfy the constraint $\sum_i MS_i e_{ij} = 0$, which arises because market share changes resulting from a change in the price of brand j must add to zero. Furthermore, $\sum_j e_{ij} = 0$, which indicates that if all brands were to change their prices by the same percentage, holding all other things fixed, market shares would be unaffected (see Equation 4).

Relationship to Brand Switching

Recall that we assumed prices and nonprice marketing activity to be essentially fixed during the hypothetical short time period. The price elasticities in the previous expressions are based on the changes in choice probabilities that would result from infinitesimal changes in prices (i.e., they are point elasticities based on derivatives, not arc elasticities). Consequently, to derive the elasticities, choice probabilities need undergo only infinitesimal changes as well. If we then condition on the prevailing set of brand prices and nonprice marketing activity during the short time period, the result is that choice probabilities remain essentially the same from one purchase to the next. Because of our assumption that the error term z_{ikt} in Equation 1 is distributed independently across purchases, each consumer's probabilistic choice behavior follows a zero-order stochastic process. (The next section considers a model in which the individual consumer's choices are correlated over time.) Thus, we can write

$$\pi_{iik} = \theta_{ik}\theta_{ik} \tag{14}$$

and

$$\pi_{ijk} = \theta_{ik}\theta_{jk} \text{ for } i \neq j, \tag{15}$$

where π_{iik} is the probability that consumer k purchases brand i in two successive occasions, and π_{ijk} is the probability that consumer k switches from brand i to brand j. Aggregating across consumers, we obtain

(16) $$\pi_{ii} = \frac{\sum_k \theta_{ik}\theta_{ik}}{N}$$

and

(17) $$\pi_{ij} = \frac{\sum_k \theta_{ik}\theta_{ik}}{N} \text{ for } i \neq j,$$

where π_{ii} and π_{ij} represent the elements of the aggregate or market-level cross-classification matrix.

The elements of the row-conditional switching matrix specify the probabilities of staying with brand i or switching to brand j, conditional on current purchase of brand i. These elements are defined to be

(18) $$r_{ii} = \frac{\pi_{ii}}{MS_i} = \frac{\sum_k \theta_{ik}\theta_{ik}}{\sum_k \theta_{ik}}$$

and

(19) $$r_{ij} = \frac{\pi_{ij}}{MS_i} = \frac{\sum_k \theta_{ik}\theta_{jk}}{\sum_k \theta_{ik}} \text{ for } i \neq j.$$

Combining Equations 12 and 13 with Equations 18 and 19 enables us to write down the relationship between the switching matrix elements and the elasticity matrix elements:

(20) $$e_{ii} = -\beta_p(1 - r_{ii})$$

and

(21) $$e_{ij} = \beta_p r_{ij} \text{ for } i \neq j.$$

The market share cross-price elasticity is thus proportional (through β_p) to the aggregate row-conditional switching probability. The market share own-price elasticity is proportional (through the negative of the same scaling constant) to one minus the corresponding row-conditional repeat purchase probability.

Discussion of the Elasticity–Switching Relationships

The elasticity–switching relationship of Equation 21 can be understood intuitively as follows: (The explanation of Equation 20 is analogous.) As was seen previously, the logit model-based cross-price elasticity e_{ijk} is equal to $\beta_p\theta_{jk}$ at the consumer level. Define r_{ijk} as the conditional probability that consumer k switches to brand j, given a previous purchase of brand i. Because the consumer choice process is zero order, $r_{ijk} = \theta_{jk}$. Clearly, the cross-elasticity e_{ijk} is proportional to the conditional switching probability r_{ijk} for an individual consumer.

Our derivation shows that this proportionality extends to the corresponding market-level measures. The market-level cross-elasticity e_{ij} is a weighted average of the consumer-level cross-elasticities e_{ijk}, with consumer weights w_{ik}. In addition, the market-level row-conditional switching probability r_{ij} can be written as a weighted average of the consumer-level conditional switching probabilities r_{ijk} ($= \theta_{jk}$) using the same weights w_{ik} (see Equation 19). Because the same consumer weights apply in both cases, the proportionality between cross-elasticities and conditional switching probabilities found at the individual level also holds at the market level. In essence, the aggregate elasticity–switching relationship is a manifestation of the fact that choice probabilities determine both elasticity and switching in a similar manner.

Although the previous derivation has been illustrated specifically for price elasticity, it is evident that analogous relationships apply for the nonprice marketing variables. The only difference lies in the proportionality constant between the switching probabilities and the elasticities. For price, this is given by β_p, the price coefficient in the choice probability model (Equation 4). For nonprice variables, this is given by the corresponding β_l. An important implication of these results is that the market share elasticity matrix for all marketing variables will share the same template but will differ only by a scaling factor and/or sign (e.g., own-price elasticities should be negative, but own-display elasticities should be positive). Thus, the greater the impact of a marketing variable on choice probability, the greater is the corresponding elasticity matrix (in absolute magnitude).

The correspondence between the brand switching probabilities and (market share) elasticities was derived by assuming that the errors z_{ikt} (Equation 1) are distributed identically and independently across brands, consumers, and choice occasions. The assumptions made on the error term are the same as those in the classical logit model. Although these error term assumptions have not been tested explicitly, we note that the overall results from the logit model appear to be quite robust given the large number of successful applications and comparative studies undertaken (e.g., Kalwani, Meyer, and Morrison 1994).

The logit model sometimes is criticized on the basis of its IIA assumption. Kamakura and Srivastava (1984) report that the IIA assumption is a reasonable one when applied at the individual consumer level (as is done here), though it is violated at the aggregate level. Stated differently, the violation of IIA at the aggregate level is often due to the heterogeneity of consumers (Hausman and Wise 1978). Because our formulation captures heterogeneity across consumers, we expect Equations 20 and 21 to be good approximations in many markets. Nevertheless, if the violation of IIA at the individual consumer level is substantial, we could improve the correspondence by viewing brands as falling into strongly defined subcategories (e.g., caffeinated versus decaffeinated coffee, diet versus regular soft drinks) and using the nested-logit model at the individual consumer level. The correspondence between switching probabilities and elasticities can be extended to such partitioned markets (details available on request). In the empirical section, we also assess the correspondence using (1) a full-covariance probit model and (2) a multisegment version of the logit model.

INTERTEMPORAL GENERALIZATION[7]

The relationship between elasticities and brand switching derived in the previous section is conditional on the brand prices and marketing activity remaining essentially fixed

[7]Readers who are not technically inclined can skip this section and proceed to the Empirical Evaluation section. The main impacts of the intertemporal generalization are that the scaling constant β_p is modified and the approximate relationship becomes more exact as the number of time periods becomes large. The latter is denoted by the symbol "plim" (probability limit).

during a short time period. In practice, however, brand switching matrices are computed over a time horizon in which brand marketing mix activities (and therefore choice probabilities) fluctuate over time.[8] Subsequently, we show that a brand switching matrix constructed by temporally aggregating the purchase histories of a consumer panel also corresponds to the market share price elasticity matrix. Once again, the logic of the price elasticity derivation will extend to the nonprice marketing variables.

Market Equilibrium

We assume that the market is in equilibrium so that there are no long-term trends in consumers' brand choice probabilities or market shares. Lal and Padmanabhan (1995) report that market shares are stationary for a majority of products in the 91 product categories they studied. Our analysis is similar to Bass and Pilon's (1980, p. 487) characterization of markets, in which "brand preferences and market shares remain in long-run equilibrium over a long period of time, but are temporarily perturbed from these long-run equilibrium values by marketing activities and other events." We believe that our approach is appropriate for mature, nondurable product categories—markets characterized by a high degree of brand switching yet with minimal changes in long-term brand shares.

Autoregressive Choice Probabilities

To operationalize the notion of market equilibrium, we assume that changes in the marketing mix elements of all brands cause a consumer's choice probabilities to fluctuate over time according to a first-order autoregressive time series process. This process has two important properties for our analysis. First, it enables choice probabilities to be correlated over time. Implicitly, these correlations in choice probabilities arise because marketing mix activity (price and other elements) can be correlated over time. Second, the time series process implies that each consumer's purchase probabilities fluctuate around a stable long-term mean. Thus, market shares (and choice probabilities) fluctuate from week to week, but long-term average market shares are stable.

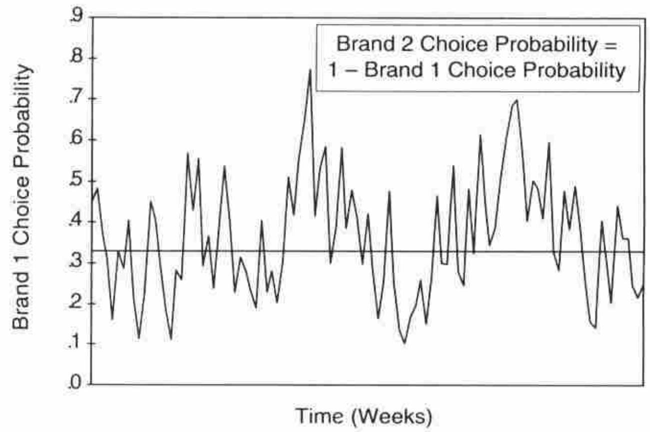
Formally, we assume that the choice probability at time period t, θ_{ikt}, can be modeled as a first-order autoregressive time series process of the form

$$(22) \qquad \theta_{ikt} = \rho\theta_{ik[t-1]} + (1-\rho)\phi_{ikt},\ t = 0, 1, 2, ..., T,$$

where $0 \leq \rho \leq 1$ and $\{\phi_{ikt}\}$ for consumer k is distributed independently and identically across time as a Dirichlet vector with means $\{\theta^*_{ik}\}$ and variability parameter α. As shown in Equation 47 of the Appendix, θ^*_{ik} is also the expected value of θ_{ikt}; that is, θ^*_{ik} is the long-term choice probability of consumer k for brand i. Figure 1 provides an illustration of Equation 22 for the special case of a two-brand market for

Figure 1

CHOICE PROBABILITY TIME SERIES



a hypothetical consumer with mean $\theta^*_1 = .333$, variability parameter $\alpha = .25$, and "stickiness" parameter $\rho = .5$. As can be seen from Figure 1, brand 1's choice probability fluctuates substantially over time (possibly because of promotional activity) and exhibits positive autocorrelation.

The first term on the right-hand side of Equation 22 states that consumer k's probability of choosing brand i at time t is related to what it was during the previous time period. Our development is consistent with models of consumer choice inertia (but not variety seeking) found in the stochastic brand choice literature (see, e.g., Givon 1984) because Equation 22 implies that a consumer's choices are likely to be correlated positively over time. The greater the parameter ρ, the greater the "stickiness." The second term ϕ_{ikt} denotes consumer k's probability of choosing brand i at time t had there been no stickiness from period to period. Intuitively, ϕ_{ikt} is the choice probability based on brand preference, price, and other marketing activity at t. Note that ϕ_{ikt} would be correlated across consumers as would be the case, for example, when a brand promotes. If marketing mix activity does not change much over time, then the variance of ϕ_{ikt} is low and the consumer's choice probabilities will be approximately constant. In general, Equation 22 implies that choice probability—and therefore an individual consumer's choice behavior—is correlated positively over time.[9]

Conditional Independence

In addition to Equation 22, we assume *conditional independence*; that is, conditional on a consumer's choice probability vectors $\{\theta_{ikt}\}$, his or her brand choices over time are assumed to be mutually independent. This assumption, which is made commonly in the context of logit-based analyses of scanner panel data, implies that any correlations in a consumer's brand choices arise from correlations in the time series of probability vectors $\{\theta_{ikt}\}$ and not from intertempo-

[8]From a competitive game-theoretic point of view, one can conceptualize each brand manager as choosing his or her price and other marketing variables to maximize profits. The resulting mixed-strategy equilibrium (e.g., Raju, Srinivasan, and Lal 1990) would mean that price and other marketing variables for each brand can be thought of as random draws from their respective probability distributions. Thus, price and other marketing variables for each brand fluctuate over time. A detailed examination of the underlying mechanisms that lead to fluctuations in marketing activities is beyond the scope of this article.

[9]As pointed out by a reviewer, there are several explanations why choice probabilities can be autocorrelated positively over time: marketing mix effects, purchase event feedback, and exogenous influences (such as weather). However, given the form of Equation 1, the only plausible candidate in our framework is temporal correlation in the marketing mix. The alternative explanations would require a reformulation of the deterministic portion of Equation 1.

ral correlations in the stochastic elements of the choice process. In essence, conditional independence allows for correlations between adjacent purchases but restricts the source of the correlations to the probability vectors $\{\theta_{ikt}\}$. Taken together, conditional independence and the time series process of Equation 22 are considerably more general than the classical, stationary zero-order choice model. Because probabilities can vary and be correlated over time but conditional independence holds, the choice process assumed here can be thought of as a *dynamic zero-order choice model.*

Temporally Aggregated Switching Matrix

The switching matrix in Equations 18 and 19, though aggregated across consumers, is not aggregated over time. Our generalization now considers the row-conditional switching matrix as is typically obtained by aggregating all adjacent brand choices of a consumer panel:

$$r_{ij} = \frac{\sum_k \sum_t Y_{ikt} Y_{jk(t+1)}}{\sum_k \sum_t Y_{ikt}}, \tag{23}$$

where Y_{ikt} is a random indicator variable (taking the value of 0 or 1) that equals 1 when consumer k buys brand i at time t. The numerator of Equation 23 gives the total number of instances (over time and consumers) in which a purchase of brand i is followed by a purchase of j. The denominator gives the total number of times brand i was purchased by the panel during the time horizon. We have written Equation 23 in a manner that implies that all consumers make a purchase in each period t = 1, 2, ..., T in a panel data set. This is done primarily for expositional purposes. The relationships between elasticities and brand switching derived subsequently are also valid in a more general case in which category purchase incidence (assumed to be exogenous) varies across consumers.

To derive the correspondence between elasticities and brand switching for the intertemporal case, we consider a long time horizon T and evaluate the probability limit (plim) of the row-conditional matrix in Equation 23. The probability limits are taken as $T \to \infty$. The number of consumers need not be large. This is given by

$$\operatorname{plim} r_{ij} = \operatorname{plim}\left[\frac{\frac{1}{T}\sum_k \sum_t Y_{ikt} Y_{jk(t+1)}}{\frac{1}{T}\sum_k \sum_t Y_{ikt}}\right] = \frac{\operatorname{plim}\frac{1}{T}\sum_k \sum_t Y_{ikt} Y_{jk(t+1)}}{\operatorname{plim}\frac{1}{T}\sum_k \sum_t Y_{ikt}}, \tag{24}$$

where the time subscript t runs over the time horizon. Using Equation 22 and the conditional independence assumption, we show in the Appendix that

$$\operatorname{plim} r_{ii} = [1 - \gamma] + \gamma R_{ii} \tag{25}$$

and

$$\operatorname{plim} r_{ij} = \gamma R_{ij},\ i \neq j, \tag{26}$$

where $\gamma = 1 - \rho\alpha(1 - \rho)/(1 + \rho)$, $(0 \le \gamma \le 1)$, and

$$R_{ij} = \frac{\sum_k \theta^*_{ik}\theta^*_{jk}}{\sum_k \theta^*_{ik}} \tag{27}$$

is the row-conditional switching matrix that would be observed if the choice probabilities of all consumers were fixed at their time series means θ^*_{ik} (see Equations 18 and 19). The parameter γ can be interpreted as one minus the correlation between Y_{ikt} and $Y_{ik[t+1]}$. Notice that $\rho = 1$ would mean that $\gamma = 1$ so that there would be no correlation between Y_{ikt} and $Y_{ik[t+1]}$ (zero-order choice behavior). This value of ρ implies constant choice probabilities (see Equation 22) and corresponds to the model discussed in the previous section.

An important implication of Equations 25 and 26 is that the switching matrix converges to a probability limit even though the choice probabilities fluctuate over time. Using algebraic manipulation, we rewrite Equations 25 and 26 as

$$1 - \operatorname{plim} r_{ii} = \gamma(1 - R_{ii}) \tag{28}$$

and

$$\operatorname{plim} r_{ij} = \gamma R_{ij},\ i \neq j. \tag{29}$$

Elasticities at Average Market Conditions

To relate the row-conditional switching matrix to elasticities, consider the average marketing environment in which each consumer k is making choices with probabilities θ^*_{ik}. Suppose that the price of one brand is changed and market share price elasticities are computed. Recall that price elasticity reflects the *marginal* effect of one brand's price holding all other prices and all marketing activity fixed. Because we are computing point elasticities, the price change considered is infinitesimal. Because this scenario is identical to the arguments leading to Equations 12 and 13 of the previous section, we can express market share price elasticities as

$$e_{ii} = -\beta_p(1 - R_{ii}) \tag{30}$$

and

$$e_{ij} = \beta_p R_{ij},\ i \neq j. \tag{31}$$

We refer to e_{ii} and e_{ij} as the "average" market share price elasticities. They are not elasticities averaged over time; instead, these elasticities characterize the market at its average conditions, that is, when consumers make choices at the respective mean choice probabilities.

Combining Equations 28 and 29 with Equations 30 and 31 enables us to express the average price elasticities as

$$e_{ii} = -\delta_p(1 - \operatorname{plim} r_{ii}) \tag{32}$$

and

$$e_{ij} = \delta_p \operatorname{plim} r_{ij},\ i \neq j, \tag{33}$$

where $\delta_p = \beta_p/\gamma$ is a proportionality constant. Equations 32 and 33 provide a simple generalization of the relationships in Equations 20 and 21 between elasticities and brand switching derived in the previous section. In comparison with the previous section, the elasticities now are computed at average market conditions, and the row-conditional switching is computed over a long time horizon. The proportionality constant $\delta_p = \beta_p/\gamma$ incorporates not only the parameter β_p, which reflects the impact of price on choice probability (Equation 4), but also the parameter $(1/\gamma)$ to inflate the obtained switching matrix $\{r_{ij}\}$ to obtain the switching matrix $\{R_{ij}\}$ at average market conditions (see Equation 26).[10] Equations 32 and 33 are similar to Equations 20 and 21 except for the proportionality constant and the consideration of a long time horizon (so that *plim* is applicable).

Intuitive Explanation of the Elasticity–Switching Relationship

We considered a realistic scenario in which prices and nonprice marketing activity vary over time so that probabilities fluctuate over time around stable means. We believe this characterizes nondurable consumer product categories that are mature. On the basis of Equation 22 and conditional independence, we show that the probability limit of the row-conditional switching matrix is a function only of the mean choice probabilities.

In the logit model, elasticities evaluated at average market conditions are solely functions of mean choice probabilities, even though they, in turn, are based on intrinsic brand preferences, prices, and other marketing variables. Consequently, to obtain elasticities at average market conditions, we merely need the mean choice probabilities. The row-conditional switching probabilities, computed over a long time horizon, provide exactly the required information on mean choice probabilities. Thus, we obtain the elasticity–switching relationships of Equations 32 and 33.

Consider a scenario in which price is considerably less important than a nonprice marketing variable, such as end-of-aisle display (β_p is much smaller than β_l). Consequently, brand switching is mostly due to end-of-aisle display and not to price. The row-conditional switching matrix—computed over a long enough time horizon—provides the information on mean choice probabilities. Because the price elasticities in the logit model are simply functions of mean choice probabilities, we are able to infer the price elasticities from the row-conditional switching matrix, even though the empirically observed switching was caused primarily by a nonprice marketing variable. In this case, we expect from Equations 32 and 33 that the magnitude of the elasticity matrix for end-of-aisle displays would be substantially higher (in absolute terms) than the elasticity matrix for price (i.e., because β_l is much larger than β_p, the proportionality constant for display, δ_l, will be much larger than the proportionality constant for price, δ_p).

EMPIRICAL EVALUATION

We begin by noting several reasons why the theoretical relationship holds only as an empirical approximation. First, Equations 32 and 33 use probability limits (plim), which implies that the row-conditional switching matrix at the individual level must be computed over many purchase occasions. In typical data sets, however, the average number of purchase occasions per panelist is not large. (In the data used here, the average is 5.3 purchases.) Second, the theory is based on point elasticities that are derived by assuming that the changes in marketing activity are infinitesimal. In practice, changes are not infinitesimal, and the resulting arc elasticities only will approximate the point elasticities. Third, the relationships relate to true values of elasticities and brand switching probabilities. In any empirical study, both constructs will be measured with error. Finally, the various assumptions made in the theoretical development would have the effect of making Equations 32 and 33 hold as approximations and not as exact relationships. Equations 32 and 33 therefore are restated as

$$e_{ii} \approx -\delta_p(1 - r_{ii}) \tag{34}$$

and

$$e_{ij} \approx \delta_p r_{ij}, \tag{35}$$

where $\delta_p = \beta_p/\gamma$. The γ parameter, interpreted as one minus the correlation between a consumer's adjacent purchases of a brand, lies between 0 and 1.

Analysis Approach

Our empirical analysis begins by computing the aggregate row-conditional switching proportions from a consumer panel using Equation 23. We then assess the quality of the approximations in Equations 34 and 35 in three ways. First, using a standard logit model, we compute aggregate own- and cross-elasticities for price, feature, and display. We examine the correlation of the switching probabilities with the corresponding elements in each estimated elasticity matrix. These correlations provide a direct evaluation of the accuracy of the theoretical relationship. Second, we compute price elasticities with the full-covariance probit model, which enables us to assess the correspondence when a model of choice behavior not subject to the IIA assumption is used to compute elasticities. Third, we estimate the logit with latent segments, which enables us to examine the correspondence when the response parameters of the logit model (i.e., the β coefficients) vary across consumer segments.

Data

The data set consists of 138 weeks of ACNielsen single-source scanner panel data on the liquid laundry detergent market in Sioux Falls, S.Dak. from 1986 to 1988. We compiled data for the nine top-selling brands (Wisk, Tide, Surf, Era, Solo, Cheer, Bold-3, All, and Fab) across the 13 stores in the data set. These nine brands captured 87% of the total market for liquid laundry detergents. The brand prices (reported subsequently) were computed as weighted averages over the four available sizes (32-ounces, 64-ounces, 96-ounces, and 128-ounces) and 13 stores. We divided the data into an initialization period (47 weeks), a calibration period (61 weeks), and a holdout period (30 weeks). (The initialization period is needed for the "loyalty" measures in the logit model.) Table 1 reports the average market shares, prices per ounce, and proportions of feature and display activities for the nine brands during the calibration period. From a panel of 2224 households, a random sample of 300 panelists who made at least four purchases per year was used to estimate the logit model. The remaining 1924 panelists were used to construct a switching matrix by counting

[10]Intuitively, the correlation over time in brand choice probabilities $\{\theta_{ikt}\}$ (induced by the temporal correlation in marketing variables) makes the switching $\{r_{ij}\}$ smaller than the switching $\{R_{ij}\}$ that would occur had consumers' choice probabilities remained constant at their long-term means $\{\theta^*_{ik}\}$.

Table 1

MARKET SHARES, PRICES PER OUNCE, AND FEATURE AND DISPLAY ACTIVITIES FOR LAUNDRY DETERGENT BRANDS

Brand	*Average Share*	*Average Price (Cents per Ounce)*	*Feature (Proportion)*	*Display (Proportion)*
Wisk	.227	4.91	.11	.25
Tide	.215	5.78	.10	.19
Surf	.195	5.22	.06	.13
Era	.135	5.84	.05	.05
Solo	.059	5.79	.02	.04
Cheer	.049	5.60	.02	.00
Bold-3	.044	5.84	.02	.01
All	.036	3.72	.00	.00
Fab	.036	5.12	.02	.00

the brand switches n_{ij} from brand i to brand j during the calibration period.[11] The row-conditional switching matrix is obtained easily as $r_{ij} = n_{ij}/\Sigma_s n_{is}$[12] and is reported in Table 2. Because different subsamples of panelists were used for estimating elasticities and brand switching probabilities, our evaluation of the relationship is somewhat conservative.

Empirical Correspondence With Logit Elasticities

Logit model estimation. Estimation follows the Guadagni and Little (1983) approach, in which the probability that consumer (or household) k selects brand i at occasion t is given by

$$\theta_{ikt} = \frac{\exp(W_{ikt})}{\sum_s \exp(W_{skt})}. \tag{36}$$

W_{ikt}, the deterministic portion of utility for brand i at choice occasion t for household k, is modeled as

$$W_{ikt} = \eta_i + \beta_1 BLOY_{ik} + \beta_2 \ln PRICE_{it} + \beta_3 FEAT_{it} + \beta_4 DISP_{it}, \tag{37}$$

where

$BLOY_{ik}$ = "loyalty" of household k to brand i;
$\ln PRICE_{it}$ = logarithm of the shelf price of brand i at occasion t;
$FEAT_{it}$ = 1 if brand i is featured at occasion t, 0 otherwise;
$DISP_{it}$ = 1 if brand i is displayed at occasion t, 0 otherwise;
$\{\eta_i\}$ = brand-specific constants to be estimated; and
$\beta_1, ..., \beta_4$ = parameters to be estimated.

Our measure of brand loyalty, $BLOY_{ik}$, is the within-panelist market share for each brand from the initialization period. Note that this differs from the exponentially smoothed measure for loyalty used by Guadagni and Little (1983) in that our measure is static (i.e., purely cross-sectional) and uses no information from the calibration period. Another difference from Guadagni and Little is our use of the logarithm of price instead of absolute price. We made both of these changes to make our estimated choice model correspond more closely to the specification in our theoretical development (see Equation 4).

We present parameter estimates and fit of the logit model of Equations 36 and 37 in Table 3.[13] All parameters are correctly signed and significant (the brand-specific constants are not presented). We also estimated the model with a linear price term and obtained virtually identical fits and nonprice parameter estimates across the two model specifications.

Elasticity computations. We next computed a matrix of own- and cross-price elasticities using the logit model, aggregating over the sample of 300 households from the panel. Following the procedures described by Guadagni and Little (1983), we lower the price of a particular brand by 1% on all choice occasions (across the 61-week period) and compute the new market shares. The own- or cross-price elasticities then are given by the percentage change in brand share from baseline share (because the price change is set to 1%). We report these in Table 4. Because the elasticities reflect response to pricing activity over the entire 61-week calibration period, they should represent average market

[11] An implication of the first-order autoregressive choice probability process of Equation 22 is that the matrix $\{n_{ij}\}$ should be symmetric aside from sampling fluctuations (see the end of the Appendix). The correlation between the string of $9 \times (9 - 1)/2 = 36$ above-diagonal elements $\{n_{ij}\}$ and the corresponding below-diagonal elements is .96.

[12] This method of constructing the brand switching matrix implicitly weights households by frequency of purchase in the category. As stated previously, there is no effect on the theoretical relationships from the differential weighting of households as long as both the switching and elasticity matrices use the same weights.

[13] We tested the IIA assumption of the logit model following the procedure described by Ben-Akiva and Lerman (1985, pp. 185–86). The test calls for comparing the fit of model parameters estimated using the full choice set against the fit of model parameters estimated using a restricted choice set. The test statistic fell short of significance at the .10 level in the tests conducted. Thus, the IIA assumption appears reasonable.

Table 2

MATRIX OF ROW-CONDITIONAL SWITCHING PROBABILITIES

	Wisk	*Tide*	*Surf*	*Era*	*Solo*	*Cheer*	*Bold-3*	*All*	*Fab*
Wisk	.481	.156	.119	.071	.022	.041	.026	.046	.039
Tide	.120	.558	.135	.055	.021	.043	.023	.020	.023
Surf	.170	.140	.424	.074	.033	.051	.030	.038	.038
Era	.102	.094	.101	.590	.027	.032	.024	.016	.014
Solo	.076	.086	.130	.057	.515	.017	.071	.017	.030
Cheer	.127	.221	.161	.085	.009	.303	.048	.030	.015
Bold-3	.138	.128	.179	.066	.072	.041	.307	.034	.034
All	.189	.130	.144	.059	.011	.048	.018	.333	.067
Fab	.132	.195	.151	.070	.026	.048	.066	.022	.290

Note: Based on brand switches of 1924 panelists in the calibration period.

Table 3

PARAMETER ESTIMATES AND FIT OF THE MULTINOMIAL LOGIT CHOICE MODEL[a]

Parameter	Coefficient Estimate	Asymptotic t-Statistic
β_1 (Brand Loyalty)	4.038	45.49
β_2 (ln Price)	−2.603	−10.71
β_3 (Feature[b])	.711	9.34
β_4 (Display[b])	.493	6.42
Log-Likelihood (LL)	−4355.9	
$\rho^2 = [1 - (LL/LL_0)]$	.244	

[a]Results of estimating the logit model of Equations 36 and 37 from the purchases of 300 panelists during the calibration period. Brand intercepts $\{\eta_i\}$ are not reported.

[b]Feature and Display are (0,1) variables. Expressing these variables in logarithmic form would not alter the results in this table; that is, coding the original dichotomous variable as (1,*e*), logarithmic transformation would give the same (0,1) variable.

conditions (as opposed to the conditions at a particular week or other short time period).

Assessment of correspondence. The empirical examination of the theory involves comparing the elements of Table 2 with the corresponding elements of Table 4. We predict that they will correspond up to a scaling constant δ_p (see Equations 34 and 35). For each diagonal entry of the switching matrix, $\{r_{ii}\}$, we transform it from r_{ii} to $-(1 - r_{ii})$ per Equation 34. We convert the elasticity matrix of Table 4 and the switching matrix of Table 2 to ($9 \times 9 = 81$ element) column vectors. Correlating the switching vector against the elasticity vector produced the following results:

Cells in Tables 2 and 4	Number of Observations	Correlation
All Cells	81	.99
Diagonal Cells Only	9	.91
Off-Diagonal Cells Only	72	.91

The large correlations (all significant at $p < .05$) strongly support the proposed theoretical correspondence for this product category. We note that the estimated elasticities in Table 4 suffer from both sampling error (300 households of 2224 in the panel) and estimation error (in the logit model parameters). The row-conditional switching probabilities in Table 2 also involve sampling error and are based on a different subsample of households. The errors induced by these factors tend to create a conservative bias in the observed correspondence between the two matrices relative to the theoretical correspondence.

We also performed a simple regression analysis of Equations 34 and 35 and obtained the following results:

Parameter	Estimate	t-ratio
Intercept	.009	.71
δ_p	3.257	56.15

The number of observations is 81 and the adjusted R^2 is .975. The intercept is not statistically significant, as predicted by Equations 34 and 35. Furthermore, as predicted by the theory, the δ_p parameter of 3.257 exceeds the β_p parameter of 2.603 in Table 3.

Nonprice marketing variables. Because newspaper feature advertising and retail display activity are both significant determinants of consumer brand choice decisions in this category (Table 3), we also computed elasticity matrices for both the feature and display variables and assessed their correspondence to the switching matrix. Both feature and display are coded in our data as dichotomous variables, that is, equal to either zero or one. To simulate a change in activity, we first determined the total number of store weeks in which features or displays were recorded during the calibration period. We then recreated a simulated marketing environment by randomly adding feature or display of the target brand to the store weeks in which it had not occurred so that feature or display activity was increased by 50%.

Using the parameter estimates in Table 3, we computed new choice probabilities and arc elasticities. To avoid chance outcomes, we repeated the computations many times and used the average elasticity values. One brand—All—had no feature activity and was dropped from that analysis. Three brands—All, Fab, and Cheer—had minimal display activity and were dropped from the display analysis. The elasticity matrices again were converted to column vectors and the correlations with the switching probabilities computed, which yielded the following:

	All Matrix Elements	Diagonal Elements	Off-Diagonal Elements
Feature Elasticities			
Correlation with Switching Matrix	.96	.86	.89
Number of Observations	72	8	64
Display Elasticities			
Correlation with Switching Matrix	.94	.78	.86
Number of Observations	54	6	48

Table 4

ESTIMATED OWN- AND CROSS-PRICE ELASTICITY MATRIX BASED ON THE MULTINOMIAL LOGIT CHOICE MODEL

	Wisk	Tide	Surf	Era	Solo	Cheer	Bold-3	All	Fab
Wisk	−1.37	.31	.37	.23	.11	.12	.09	.08	.07
Tide	.33	−1.39	.37	.23	.11	.13	.09	.08	.07
Surf	.48	.46	−1.91	.33	.16	.16	.12	.11	.09
Era	.36	.33	.39	−1.57	.11	.12	.10	.08	.07
Solo	.36	.34	.41	.25	−1.78	.13	.11	.10	.07
Cheer	.46	.47	.47	.31	.15	−2.20	.12	.13	.09
Bold-3	.49	.44	.49	.34	.18	.16	−2.32	.11	.10
All	.48	.46	.50	.32	.17	.20	.12	−2.36	.10
Fab	.50	.50	.49	.33	.16	.17	.14	.12	−2.41

Note: Price changes on column brands affect market shares of row brands.

These correlations indicate that the correspondence between switching probabilities and elasticities also holds for nonprice marketing variables in this category. The correlations, however, are somewhat lower than those for price, which could be due to several factors. First, there might be some brand-specific differences in the effectiveness of feature and display activity (recall in Equation 1 we assumed that the coefficients for marketing variables are the same across brands). Second, our measures for feature and display activity (0/1) are crude and might mask differences in the size of feature advertisements or in-store displays (e.g., larger brands might get bigger features and/or displays). Finally, the 0/1 nature of feature and display coarsens the correspondence. (Recall that infinitesimal changes were used in the theoretical derivation of elasticities.)

Robustness of the Empirical Correspondence to Violations of the IIA Assumption

An important question is whether the correspondence is empirically robust when elasticities are computed using models with less restrictive assumptions about the error term, for example, the IIA property. To assess this, we estimated a full covariance probit choice model and calculated arc price elasticities. (Unlike the logit model, the probit model assumes the error z_{ikt} in Equation 1 to be normally distributed, with potentially unequal variances across brands and the errors being potentially correlated across brands.) We used the Clark approximation to calibrate the probit model on the same 300 panelists used in the logit estimation (Table 3). The full specification of the variance–covariance matrix (with the appropriate identifying restrictions) adds 35 parameters over the logit model, and the calibration fit improves to a log-likelihood function value of –4350.8 (compared with the –4355.9 for the logit model of Table 3). As in the logit price elasticity case, arc elasticities were computed on the basis of 1% price changes.

The empirical correspondence between the switching probabilities and the probit price elasticities was examined by again forming column vectors and computing correlations:

	Number of Observations	*Correlation for Probit*
All Cells	81	.97
Diagonal Cells Only	9	.94
Off-Diagonal Cells Only	72	.82

The high correlations (all statistically significant at $p < .05$) indicate that the correspondence between switching probabilities and price elasticities is, for this data set, robust to the error term assumptions.

Robustness to the Assumption of Homogeneity of Market Response Coefficients

The specification on which our theoretical derivation is based assumes that all individuals share the same market response (β) coefficients. Several studies, however, have shown that allowing cross-sectional heterogeneity in response parameters yields significant improvements in model fit and predictive validity (e.g., Gonul and Srinivasan 1993; Jain, Vilcassim, and Chintagunta 1994; Kamakura and Russell 1989). Therefore, we further investigated the robustness of the correspondence by estimating a latent class version of the logit model of Equation 37 on the same 300 panelists.

To determine the best latent class representation of the data, we fit models involving two, three, four, five, and six latent segments. For model selection, we employed the Bayesian Information Criterion (BIC). The results indicated that the five-segment solution provided the best representation with a log-likelihood of −3864.7, a BIC of −4120.3, and 64 parameters. The six-segment solution, with 77 parameters, had a log-likelihood of −3826.1 and a BIC of −4133.7. (All models were estimated with multiple starting values to minimize the possibility of local optima.) Using the parameters from the five-segment model, we computed the own- and cross-price elasticities by obtaining baseline shares for each segment and weighting the shares by the probabilities of segment membership. The price of each brand then was lowered by 1%, and the shares were recomputed. We then correlated the price elasticities and the corresponding switching probabilities and obtained the following results:

	Number of Observations	*Correlation for Latent Class Model*
All Cells	81	.97
Diagonal Cells Only	9	.90
Off-Diagonal Cells Only	72	.84

These correlations are somewhat smaller than the ones based on the standard logit model reported previously. All the correlations, however, remain large and are statistically significant at $p < .05$. Thus, we find that the correspondence is robust to the assumption of homogeneity in the market response coefficients for the product category.

PRACTICAL USE OF THE CORRESPONDENCE

We now provide an illustration of the practical usefulness of the theoretical correspondence between switching proportions and elasticities. Suppose we wish to develop an aggregate market share model for the liquid detergent brands using store-level market share data. (Such data were also available from the same ACNielsen single-source data used previously.) To do this, we use the multiplicative competitive interaction (MCI) market share attraction model (Cooper and Nakanishi 1988), which has the advantage of imposing logically consistent constraints so that market shares sum to one. In the MCI model, market share is given by

$$(38)\qquad MS_{it} = \frac{A_{it}}{\sum_{j=1}^{m} A_{jt}},$$

where MS_{it} is the market share of brand i at time t, A_{it} is the "attraction" of brand i at time t, and m is the total number of brands. The attraction A_{it} can be specified as

$$(39)\qquad A_{it} = \exp\left(\psi_i + e_{ii} \ln p_{it} + \sum_{j \neq i} e_{ij} \ln p_{jt}\right),$$

where $e_{ii} < 0$, and $e_{ij} > 0$ for $j \neq i$ with identifying restrictions $\Sigma_i MS_i e_{ij} = 0$ for $j = 1, 2, ..., m$.[14,15] Using log-centering transformations[16] on the dependent variable $\{MS_{it}\}$, the parameters of the model in Equation 39 can be estimated directly by least-squares regression[17] (for details, see Cooper and Nakanishi 1988). The log-centering transformation used for the estimation implies that the parameters e_{ii} and e_{ij} are own- and cross-price elasticities, respectively, under average market conditions.

We assess whether the estimation of the MCI model in Equations 38 and 39 can be improved by using the correspondence of 34 and 35 between elasticities and row-conditional switching proportions. The latter are available in the present context in the consumer panel data. (As was mentioned in the introduction section, switching proportions might need to be collected in other contexts from survey data, because panel data might be unavailable.)

We estimate the MCI model in Equations 38 and 39 with and without imposing the restrictions of Equations 34 and 35. The unrestricted model estimates nine own-elasticities e_{ii} and $9 \times 8 = 72$ cross-elasticities e_{ij} (in addition to the brand intercepts ψ_i). The switching-based model, however, uses the available switching proportions $\{r_{ii}\}$ and $\{r_{ij}\}$. Substituting Equations 34 and 35 into Equation 39, we obtain the following:

$$(40)\quad A_{it} \simeq \exp\left\{\psi_i - \delta_p\left[(1 - r_{ii})\ \ln p_{it} - \sum_{j \neq i} r_{ij}\ \ln p_{jt}\right]\right\}.$$

Consequently, the switching-based MCI model (Equations 38 and 40) estimates only the single scaling parameter δ_p (in addition to the brand intercepts ψ_i).

Results

We compared the unrestricted and the switching-based models for the calibration (61 weeks) and holdout periods (30 weeks). Following, we report the number of estimated nonintercept parameters, κ, and two basic measures of fit between predicted and actual market shares: mean absolute deviation (MAD) and root mean squared error (RMSE).

		Calibration Period		*Holdout Period*	
Model	κ	*MAD*	*RMSE*	*MAD*	*RMSE*
Unrestricted MCI (Equation 39)	81	.022	.038	.036	.056
Switching-Based MCI (Equation 40)	1	.030	.050	.027	.041

[14]MS_i denotes the average market share of brand i during the calibration period.

[15]It may appear that Equation 39, which uses all brands' prices, is inconsistent with the consumer model of Equation 1, which uses only brand i's price in determining the utility (attraction) of brand i. Likewise, the model in Equation 39 uses different price coefficients for different brands rather than a common price coefficient across brands as was done in Equation 1. Note, however, that Equation 1 is an individual-level model, whereas Equation 39 is an aggregate model.

[16]We defined the log-centering using a weighted geometric mean with weights equal to average market shares during the calibration period. Details are available on request.

[17]We estimated the models using "seemingly unrelated regressions" with first-order autoregressive error on the time series of weekly market shares for each brand.

As expected, the unrestricted model, which fits many more parameters, provides the better in-sample fit. In the holdout period, however, the performance of the unrestricted model deteriorates substantially, and the switching-based restricted model outperforms the unrestricted model.[18]

Not shown in the previous result is that 26 of the 72 (36%) cross-elasticities in the unrestricted model were negative, that is, incorrectly signed. Presumably, this is due to the severe multicollinearity problems posed by the model of Equation 39. In contrast, the cross-elasticities estimated by the switching-based model all were signed correctly. (Because the switching proportions are necessarily non-negative and the estimated δ_p is positive, the cross-elasticities obtained from Equation 35 are all non-negative.)

An alternative explanation for the improved predictive validity of the switching-based aggregate model is that it is parsimonious; it estimates merely one (nonintercept) parameter compared with 81 (nonintercept) parameters of the unrestricted model (Hagerty and Srinivasan 1991). To examine this alternative explanation, we also estimated (1) a simple effects MCI model, which posits $A_{it} = \exp(\psi_i - \xi \ln p_{it})$, thereby estimating only one (nonintercept) parameter, and (2) a differential effects MCI model, which posits $A_{it} = \exp(\psi_i - \xi_i \ln p_{it})$, thereby estimating nine (nonintercept) parameters, one corresponding to each brand. The results are reported as

		Calibration Period		*Holdout Period*	
Model	κ	*MAD*	*RMSE*	*MAD*	*RMSE*
Simple Effects	1	.058	.083	.061	.080
Differential Effects	9	.035	.052	.031	.042

As seen from these results, the simple effects model performs poorly. Although the differential effects model and the switching-based MCI model (in the previous illustration) have similar forecast errors in the holdout period, the switching-based MCI model does not lead to the restrictive elasticity pattern of the differential effects model (the cross-elasticity of brand j's price change is identical across all competing brands). Thus, the switching-based MCI model forecasts well and has stronger face validity for managers.

To summarize, the switching-based elasticities yield good predictive validity in tracking market share variations induced by price changes. The switching-based elasticities are likely to be more credible to marketing managers than the often incorrectly signed cross-elasticities obtained by the unrestricted MCI model and the restricted pattern of elasticities imposed by the differential effects model. Taken overall, the empirical results for this category indicate that the theoretical correspondence is useful in estimating aggregate-level market share models.

CONCLUSION

Our objective has been to show that market share elasticities are related directly to row-conditional brand switching probabilities under a set of plausible assumptions. We showed that aggregate market share cross-elasticities are

[18]The slight reductions in MAD and RMSE in the holdout period for the switching-based models could be due to smaller price variations in the holdout period compared with the calibration period, thereby making the prediction task easier for the holdout period.

proportional (through a scaling constant) to aggregate row-conditional switching probabilities, and aggregate market share own-elasticities are proportional (through the negative of the same scaling constant) to one minus the aggregate repeat purchase probabilities.

To derive this correspondence, we used a two-phase approach. In the first phase, we held marketing activity to be essentially fixed over a short time period and considered only the infinitesimal changes needed for point elasticities. Taking a population of consumers heterogeneous in their intrinsic brand preferences, we made the standard logit assumption that consumers follow the logit choice rule at the individual level (where the error in utilities is distributed independently and identically across consumers, brands, and choice occasions).

In the second phase, we allowed prices and nonprice marketing activity to vary over time and considered the case in which consumers' choice probabilities fluctuate according to an autoregressive time series process. We showed that the temporally aggregated row-conditional switching matrix is proportional to the own- and cross-elasticities evaluated at average market conditions. The correspondence between elasticities and row-conditional switching probabilities is obtained because (1) elasticities in the logit model are solely functions of choice probabilities, even though the choice probabilities, in turn, depend on intrinsic brand preferences, prices, and other marketing variables, and (2) brand switching is also a function of choice probabilities.

We conducted an empirical evaluation of the correspondence using scanner panel data for the liquid laundry detergent category. We computed a row-conditional switching matrix and compared this with three own- and cross-elasticity matrices (for price, feature, and display) based on a logit model for a separate set of panelists. The correlation between the switching probabilities and the corresponding estimated elasticity matrix elements was high in each case. We also assessed the correspondence between the switching probabilities and the price elasticities based on the full-covariance probit and again found a high correlation. Finally, we showed that the correspondence is robust when we allowed for variations in response coefficients across consumer segments. Overall, our results indicate that the theoretical relationship provides a good approximation for the laundry detergent data. Needless to say, it would be worthwhile to examine the relationship for several other product categories.

Implications of the Findings

The correspondence between elasticities and brand switching should be of value to both researchers and managers. From a theoretical perspective, the research provides a simple link between two constructs that historically have been treated separately in the marketing literature—the older market structure literature based on brand switching and the newer market structure literature based on cross-price elasticities. For example, consider the Hendry definition of a market partition: brands fall into the same partition if aggregate switching is in proportion to market share ($r_{ij} = \tau MS_j$). If a switching matrix follows the Hendry pattern, then our theory implies that cross-price elasticities must follow a simple proportional draw structure ($e_{ij} = \xi MS_j$). In general, the linkage between market structure and elasticity patterns should be useful in developing realistic demand models in applied settings.

If household-level scanner panel data are available with store environment information, then elasticities should be computed by logit models, as in the empirical section. Unfortunately, data on household purchase histories are not available (or at best are incomplete) for some product categories and, even if available, only cover restricted geographic areas (Russell and Kamakura 1994). Moreover, the increasing use of "wand" or "Handiscan" panels creates situations in which store environment data are too spotty to provide reliable information for the logit analysis. In these same situations, however, the researcher might have access to reliable information on observed brand switching (e.g., current and previous purchase) or can use survey data.

In such data-poor situations, our theory provides an approach to estimate cross-elasticities. Row-conditional switching probabilities provide the elasticity estimates except for a different multiplicative scaling constant for each marketing variable. The scaling constants can be estimated by an econometric analysis of market-level data (which are almost always available) by relating market shares to marketing variables as in the previous illustration of the switching-based MCI model. As shown in that case, the switching-based approach is likely to predict better than a direct estimation of all brands' own- and cross-elasticities by econometric methods. In our empirical work, multicollinearity problems resulted in inappropriate (negative) signs for more than one-third of the cross-elasticities estimated by the direct approach. In contrast, the use of brand switching information forces cross-elasticities to reflect the observed substitutability of the brands under consideration.

From a managerial perspective, the obtained theoretical correspondence is of significant assistance in contexts in which brand switching proportions are available but price elasticities are not. For example, the manager of brand j considering a price decrease would need to anticipate those competitive brands that are more likely to react to his or her price decrease. Using cross-price elasticity as an indicator of potential competitive reaction, we suggest identifying those brands with the largest numbers in column j of the row-conditional switching matrix. Likewise, the manager of brand i would like to identify those competitive brands to whose promotions his or her brand is most vulnerable. Our results suggest paying greater attention to those brands with the largest numbers in row i of the row-conditional brand switching matrix.

Limitations and Extensions

Several limitations of the current theory should be noted, some of which are also avenues for further research. In our theoretical derivation, we assume that the coefficients of marketing variables in the logit model do not vary across consumers. As noted previously, this assumption does not imply that market share elasticity is identical across consumers. The assumption, however, does imply that if one customer is more responsive than another for a given marketing instrument, then he or she is also more responsive to other marketing instruments. We hope that further research will extend our results by allowing a greater degree of heterogeneity in response parameters (e.g., Gonul and Srinivasan 1993; Jain, Vilcassim, and Chintagunta 1994; Kamakura and Russell 1989).

Our analysis also assumes that the various brands in the product class are substitutes. In some categories, a subset of the brands might be complements (e.g., different flavors of a yogurt manufacturer's product line). In addition, extending the current theory beyond brand choice to include the effects of marketing activity on purchase incidence and purchase quantity would be worthwhile. This would permit our results on market share elasticities to be extended to sales elasticities.

Research is under way to use the present theory to develop a prior distribution for price elasticities. The researcher then could construct empirical Bayes estimates as a compromise between the values implied by an unconstrained econometric analysis on store-level data and the values implied by brand switching data. The empirical Bayes method also would have the advantage of effectively combining two different components of "single-source" market research data: brand switching obtained from consumer panel data merged with elasticity estimates obtained from store-level data. The connection between the two key measures of interbrand substitution also should enable researchers to begin linking together market structure approaches based on brand switching with approaches based on price elasticities.

APPENDIX: PROBABILITY LIMIT OF THE SWITCHING MATRIX[19]

On the basis of the assumptions about the indicator variables Y_{ikt} (see Equation 23), it follows that

$$E[Y_{ikt}] = E[\theta_{ikt}] \tag{41}$$

and

$$E[Y_{ikt}Y_{jk(t+1)}] = E[\theta_{ikt}\theta_{jk(t+1)}], \tag{42}$$

where $E[\cdot]$ denotes the expectation operator. The second equation follows from the conditional independence assumption; that is, conditional on a consumer's choice probability vectors $\{\theta_{ikt}\}$, his or her brand choices over time are assumed to be mutually independent. We proceed by evaluating these expressions and using the results to compute plim r_{ij}.

Consider the autoregressive time series process given in Equation 22. Let $\phi^*_{ikt} = \phi_{ikt} - \theta^*_{ik}$, where ϕ_{ikt} is one element of a Dirichlet distributed vector with mean $\{\theta^*_{ik}\}$ and scale parameter α. Consequently,

$$E[\phi^*_{ikt}] = 0, \tag{43}$$

$$E[\phi^*_{ikt}\phi^*_{ikt}] = \alpha\theta^*_{ik}(1 - \theta^*_{ik}), \tag{44}$$

$$E[\phi^*_{ikt}\phi^*_{jkt}] = -\alpha\theta^*_{ik}\theta^*_{jk},\ i \neq j, \tag{45}$$

and

$$\theta_{ikt} = \theta^*_{ik} + (1 - \rho)\sum_{s=0}^{\infty}\rho^s\phi^*_{ik(t-s)}. \tag{46}$$

Equations 43 and 46 imply that

$$E\left[\theta_{ikt}\right] = \theta^*_{ik}. \tag{47}$$

[19]Details of the proofs in the Appendix are available on request.

Consider the denominator of plim r_{ij} (Equation 24). From Equations 41 and 47, it takes the form

$$\text{plim}\frac{1}{T}\sum_k\sum_t Y_{ikt} = \sum_k \text{plim}\frac{1}{T}\sum_t Y_{ikt} = \sum_k \theta^*_{ik}. \tag{48}$$

Because ϕ_{ikt} (and therefore ϕ^*_{ikt}) are assumed to be distributed independently across time, it follows that $E\left[\phi^*_{ikt}\phi^*_{jk(t+s)}\right] = 0$ for $s \neq 0$. It then follows from Equation 22 that

$$E\left[\theta_{ikt}\theta_{jk(t+1)}\right] = \rho E(\theta_{ikt}\theta_{jkt}) + (1 - \rho)\theta^*_{ik}\theta^*_{jk}. \tag{49}$$

Using Equations 45 and 46, we get

$$E\left[\theta_{ikt}\theta_{jkt}\right] = \left[\frac{1 - \alpha(1 - \rho)}{(1 + \rho)}\right]\theta^*_{ik}\theta^*_{jk},\ i \neq j. \tag{50}$$

From Equations 49 and 50,

$$E\left[\theta_{ikt}\theta_{jk(t+1)}\right] = \gamma\theta^*_{ik}\theta^*_{jk},\ i \neq j, \tag{51}$$

where $\gamma = 1 - \rho\alpha(1 - \rho)/(1 + \rho)$. Similar reasoning yields

$$E\left[\theta_{ikt}\theta_{ik(t+1)}\right] = (1 - \gamma)\theta^*_{ik} + \gamma\theta^*_{ik}\theta^*_{ik}. \tag{52}$$

Consider the numerator of plim r_{ij} (Equation 24): From Equation 42, we obtain

$$\text{plim}\frac{1}{T}\sum_k\sum_t Y_{ikt}Y_{jk[t+1]} = \sum_k \text{plim}\frac{1}{T}\sum_t Y_{ikt}Y_{jk[t+1]} = \sum_k E\left[\theta_{ikt}\theta_{jk[t+1]}\right]. \tag{53}$$

Using Equations 51 and 52, we obtain

$$\text{plim}\frac{1}{T}\sum_k\sum_t Y_{ikt}Y_{ik[t+1]} = (1 - \gamma)\sum_k \theta^*_{ik} + \gamma\sum_k \theta^*_{ik}\theta^*_{ik} \tag{54}$$

and

$$\text{plim}\frac{1}{T}\sum_k\sum_t Y_{ikt}Y_{jk[t+1]} = \gamma\sum_k \theta^*_{ik}\theta^*_{jk},\ i \neq j. \tag{55}$$

Equation 55 implies that the probability limit of the cross-classification matrix is symmetric. Taking the ratio of Equations 54 and 55 to Equation 48 establishes the probability limits in Equations 25 and 26 stated previously.

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